Recent Monte Carlo Results for Cold Atoms

- Introduction
  - Equal Mass at Unitarity
  - Improved DMC results
  - Initial lattice results
- Unequal Masses
  - Superfluid state
  - Normal (Polarized) state
  - Few-Particle states
- Future possibilities

Stefano Gandolfi (LANL), Alex Gezerlis (LANL/UW), Michael Forbes (UW)

Shiwei Zhang (W&M), Kevin Schmidt (ASU)

Joaquin Drut (LANL)

Cold Atoms near Unitarity

Continuum

$$H = \sum_{i=1,n_l} \frac{-\hbar^2}{2m_l}\nabla_i^2 + \sum_{j=1,n_h} \frac{-\hbar^2}{2m_h}\nabla_j^2 + \sum_{i,j} V(r_{ij})$$

$$v(r) = -\frac{2}{m}\frac{\mu^2}{\cosh^2(\mu r)}$$

Take the limit $\mu \to 0$

Single scale in the problem: $k_F = (3\, \pi^2\, \rho)^{1/3}$

$E = \xi\, E_{FG} = \xi\, (3/5)\, k_F^2/\, (2m)$

$\Delta = \delta\, E_F = \delta\, k_F^2/\, (2m)$

$C = 8\, \pi^2\, \rho^2\, A^2 = \varsigma\, (2\, k_F^4\, /\, (5\, \pi))$

$g(r) \to A^2/\, r^2$

Rich Experimental Control and Probes:
phase diagram, `exotic' superfluids, RF response,...

Equal Masses: Improved DMC calculations

Forbes, Gezerlis, Gandolfi (2010)
Gandolfi, Schmidt, Carlson (2010)

Upper Bound to the Energy
Applicable to polarized, unequal mass,...

$$\begin{aligned} \Psi_T &= \prod_{ij} f_{ij'} \Phi_{\rm BCS} \\ \Phi_{\rm BCS} &= \mathcal{A}[\phi(r_{11'})\phi(r_{22'})...\phi(r_{nn'})] \end{aligned}$$

Canonical Ensemble
Dilute
Periodic Boundary Conditions

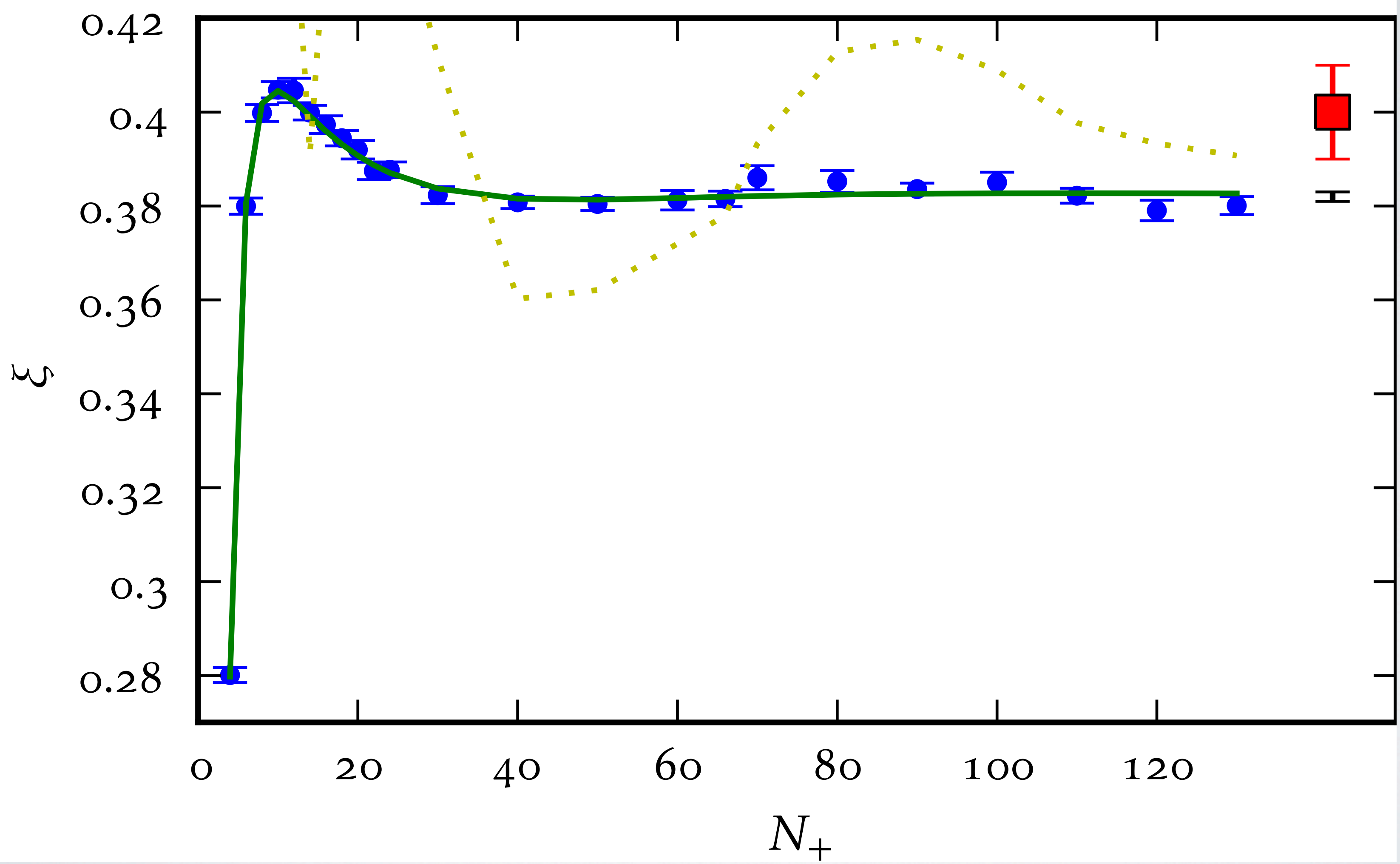
$$\begin{aligned} \phi(\mathbf{r}) &= \tilde{\beta}(r) + \sum_n a(k_n^2) \exp[i\vec{k}_n \cdot \vec{r}] \ , \\ \tilde{\beta}(r) &= \beta(r) + \beta(L-r) - 2\beta(L/2) \ , \\ \beta(r) &= [1+cbr]\ [1-\exp[-10br]] \frac{\exp[-br]}{br} \end{aligned}$$

Pair function
optimized variationally

$\frac{L^2}{4\pi^2}k^2$	$a(k^2)$	$\frac{L^2}{4\pi^2}k^2$	$a(k^2)$
0	0.00198	5	0.000190
1	0.00250	6	0.000200
2	0.00194	8	0.000167
3	0.00081	9	0.000163
4	0.00033	10	0.000120

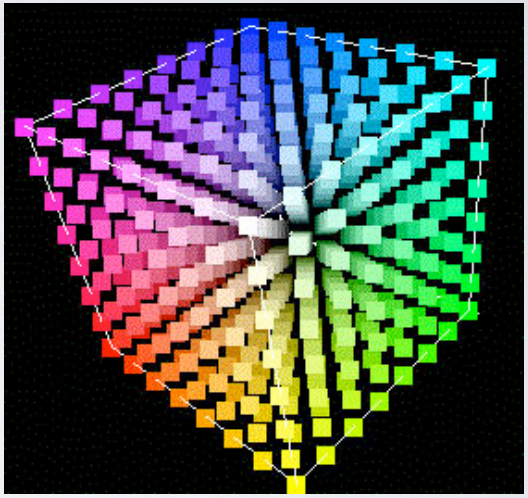
Finite Volume Effects

Forbes, Gezerlis, Gandolfi (2010)



Lattice Approaches (in progress)

Equivalent to attractive Hubbard model in dilute limit
No sign problem, but dilute limit non-trivial
Canonical approach (more efficient for T=0)



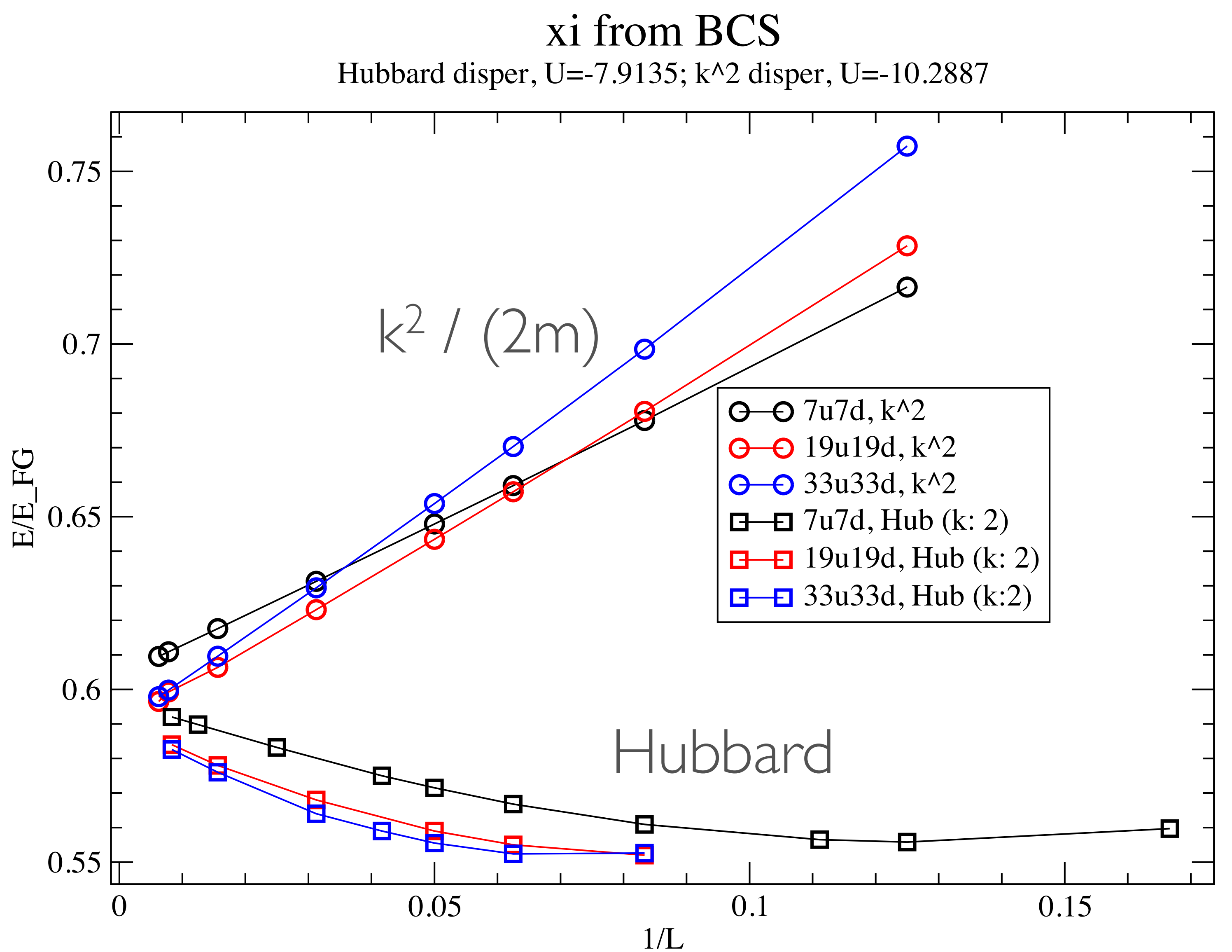
Evolve N single-particle wave functions w/ exp [- H τ]
Kinetic Energy diagonal in momentum space
Interaction set in auxiliary fields, tuned to give
zero-energy bound state on infinite lattice
Auxiliary fields for interaction sampled by MC

Different operators lead to same continuum result
Simplest Interaction is the Hubbard Model:
    On-site repulsion
    Nearest-neighbor hopping

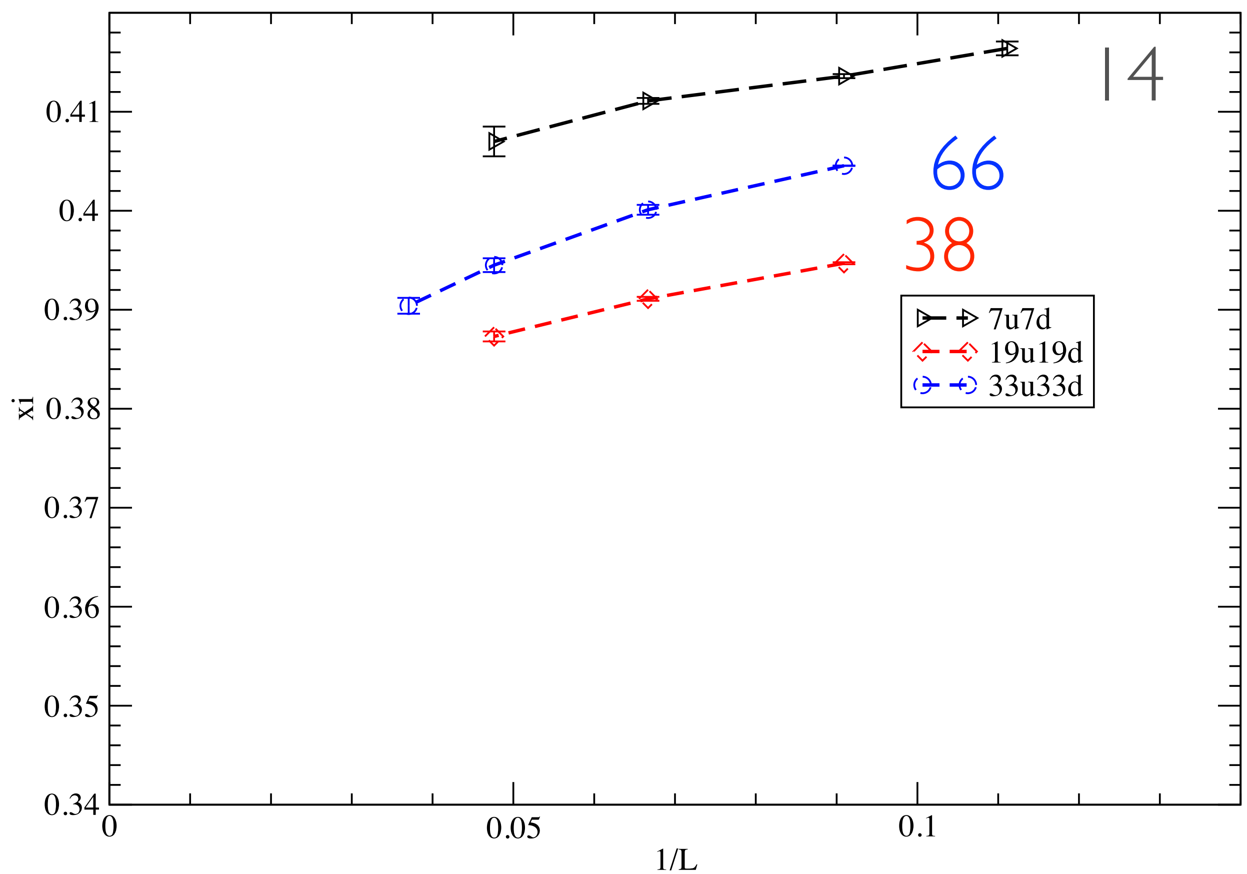
Example: Kinetic Energy

Hubbard model (nearest neighbor hopping)
$k^2 / (2m)$ (easily evaluated via FFT)
$+ O(k^4)$ match 2-body spectra (effective range)

BCS on the lattice

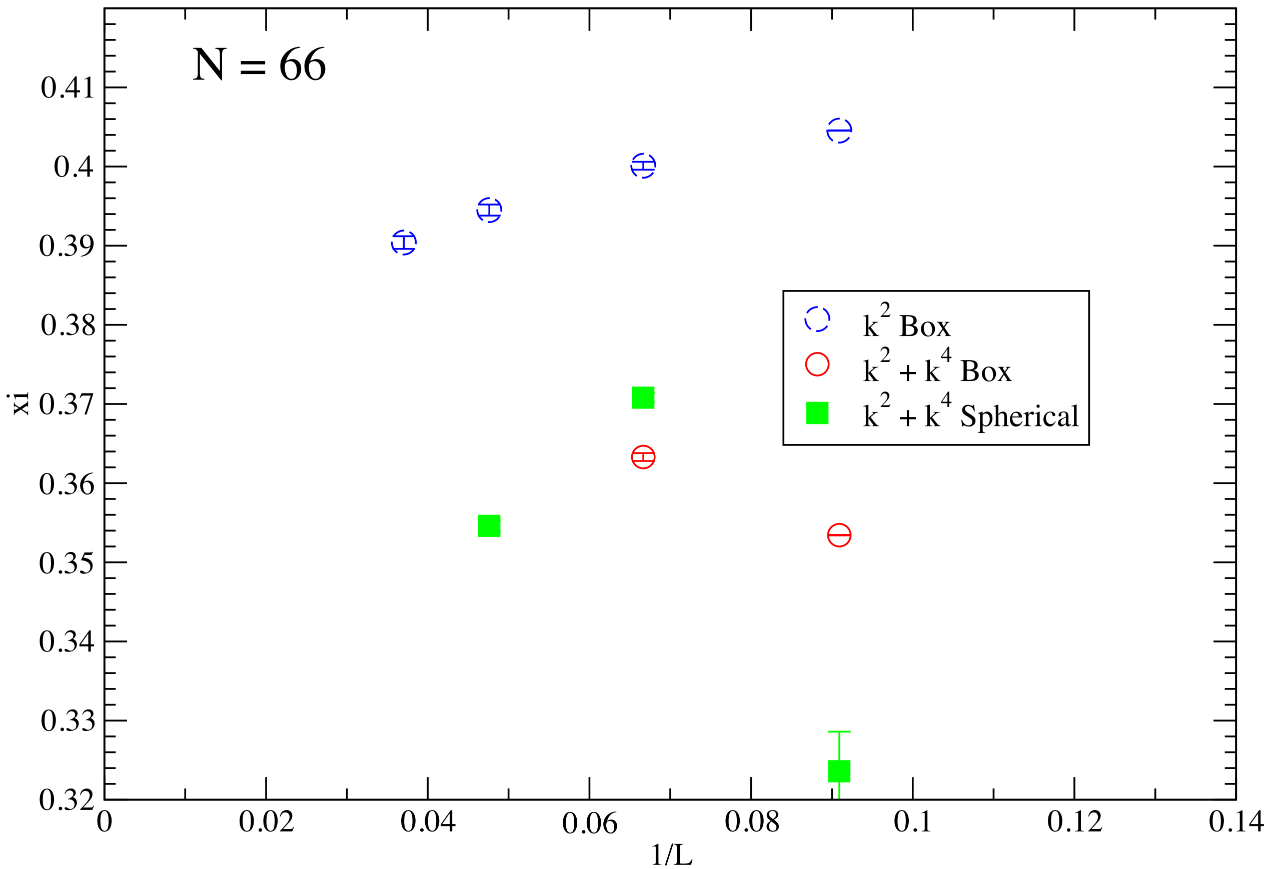


Preliminary Results



Energy vs. L for different N: k^2 dispersion

Preliminary Results



Energy vs L for different dispersions, N = 66

Contact (Shina Tan)

Controls short-distance (high-momentum) dynamics

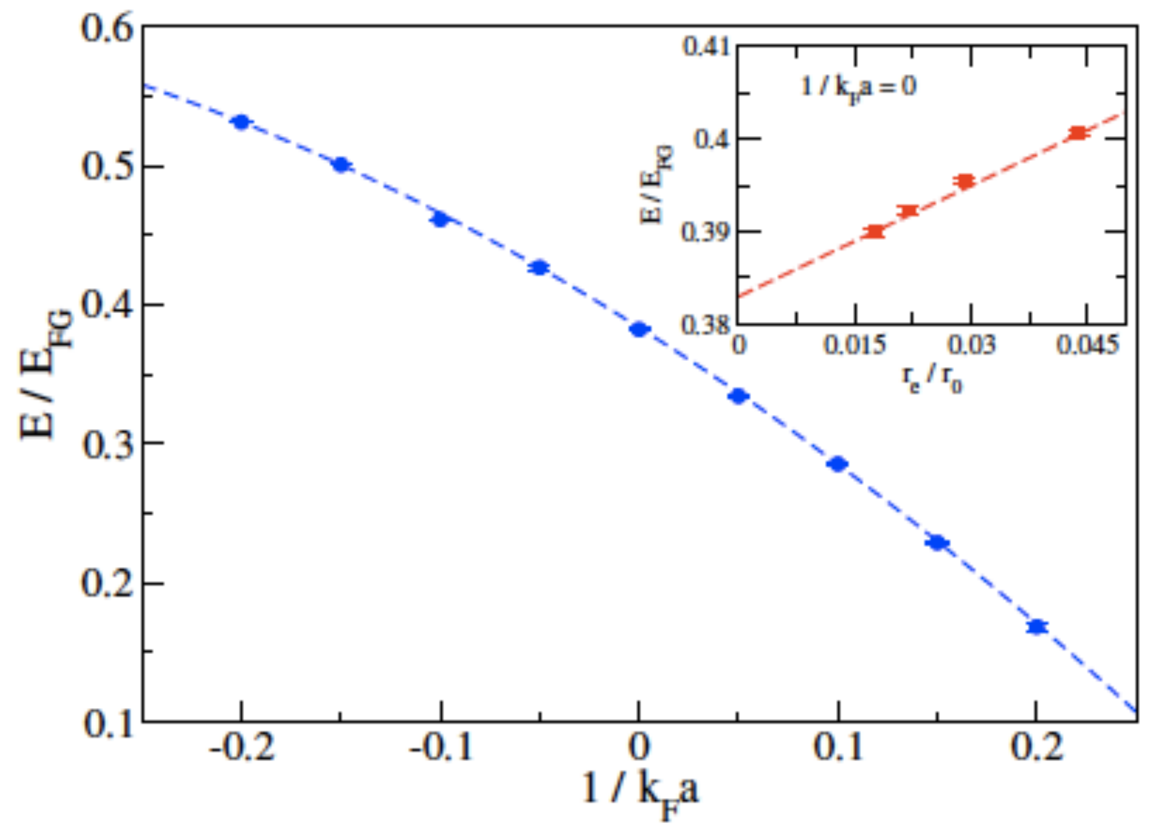
$\psi \rightarrow \alpha/r$

$n(k) \rightarrow 1/k^4$

Since interaction is zero-ranged, can be measured in EOS
Also controls high-energy part of RF response

Contact and the EOS near unitarity

Gandolfi, Schmidt, Carlson (2010)



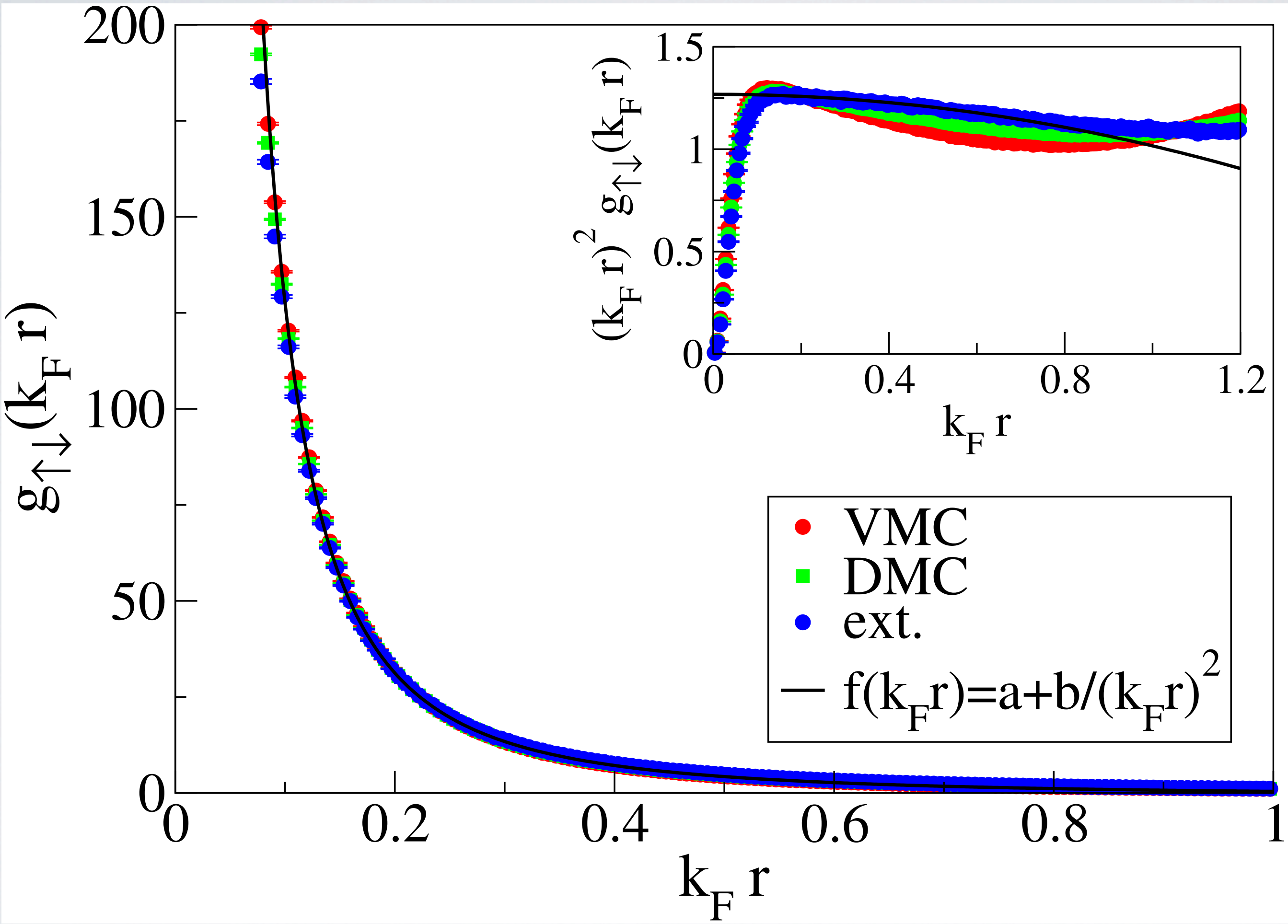
uses Feynman-Hellman Theorem

$$\frac{E}{E_{FG}} = \xi - \frac{\zeta}{k_F a} - \frac{5\nu}{3(k_F a)^2} + \dots ,$$

$\xi = 0.383(1)$ $\zeta = 0.901(2)$

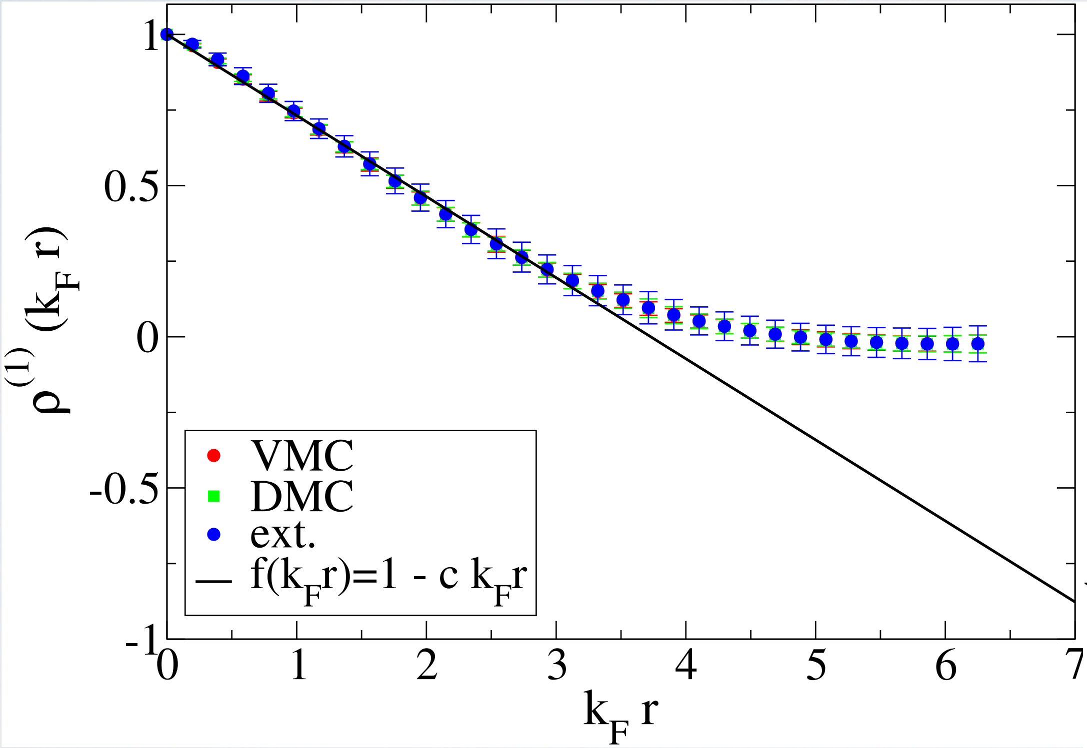
$$\frac{C}{k_F^4} = \frac{2\zeta}{5\pi} = 0.1147(3)$$

Contact and the Pair Distribution Function

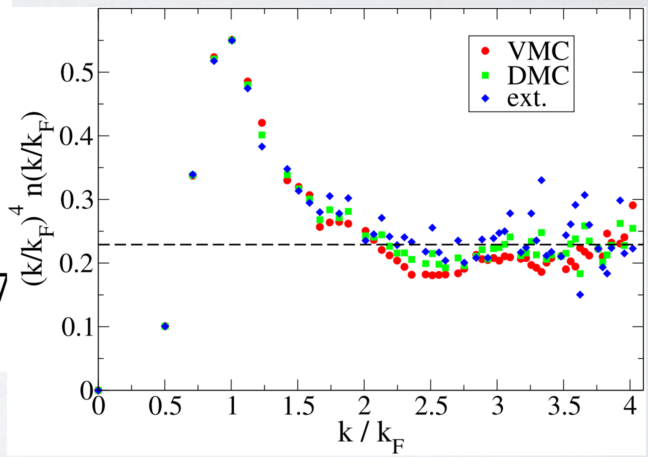


$$g_{\uparrow\downarrow}(r) \rightarrow \frac{9\pi}{20}\zeta(k_F r)^{-2} \qquad \zeta = 0.897(2)$$

Contact and the off-diagonal one-body density matrix



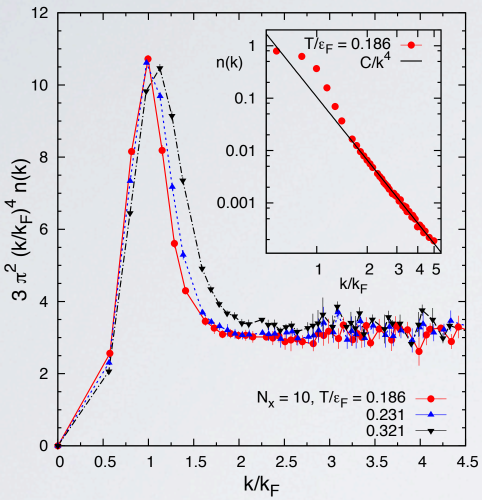
Momentum Distribution

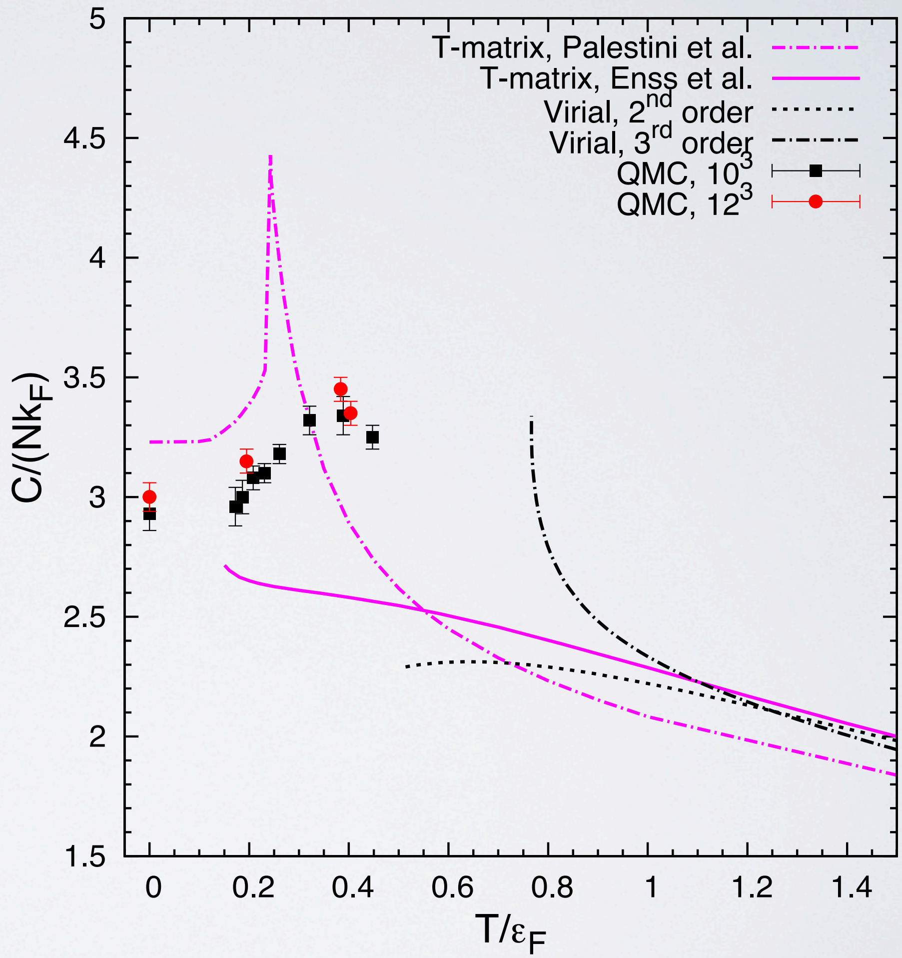


$$S_{\uparrow\downarrow}(k) - \frac{1}{2} = \frac{2\pi^2 n A^2}{k}\left[1 - \frac{1}{4\pi a k}\right] + \ldots \qquad \zeta = 0.895(16)$$

Contact at Finite Temperature

Drut, Lahde, and Ten (2010)





Increase near T=0 predicted by phonons; fermion nature at large T

Unequal Masses

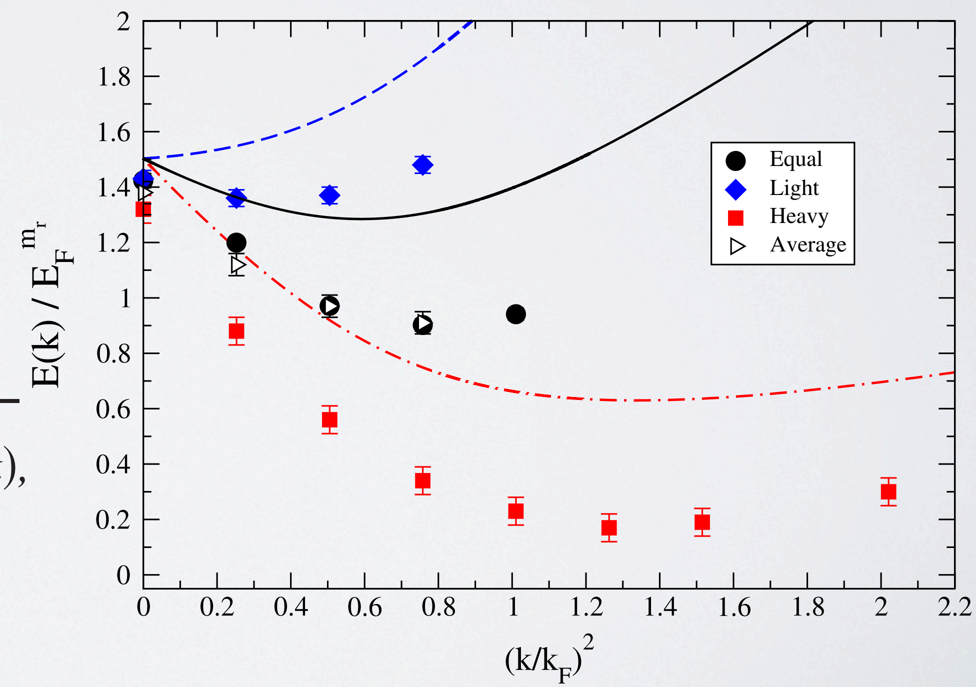
Additional parameter to probe for new physics

BCS solution unchanged for different reduced mass
Equal mass solution good starting point

ξ (M/m = 6.5) - ξ (M/m = 1) = -0.02

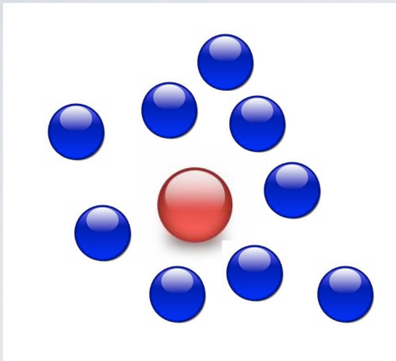
In BCS theory

$$E_{h(l)}(k) = \frac{\xi_{h(l)}(k) - \xi_{l(h)}(k)}{2} + \sqrt{\left(\frac{\xi_h(k) + \xi_l(k)}{2}\right)^2 + \Delta^2(k)},$$



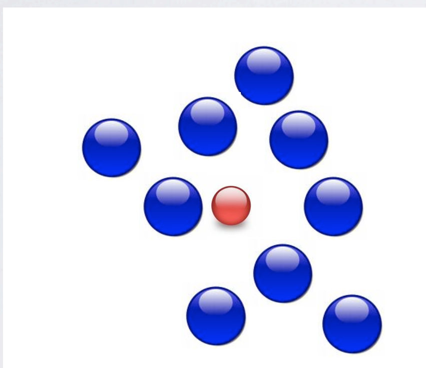
Gandolfi, Gezerlis, Carlson, Schmidt (2009)

Binding of One Heavy or One Light



$B(H) = 0.36\ E_F(L)$

effective mass ~1.0



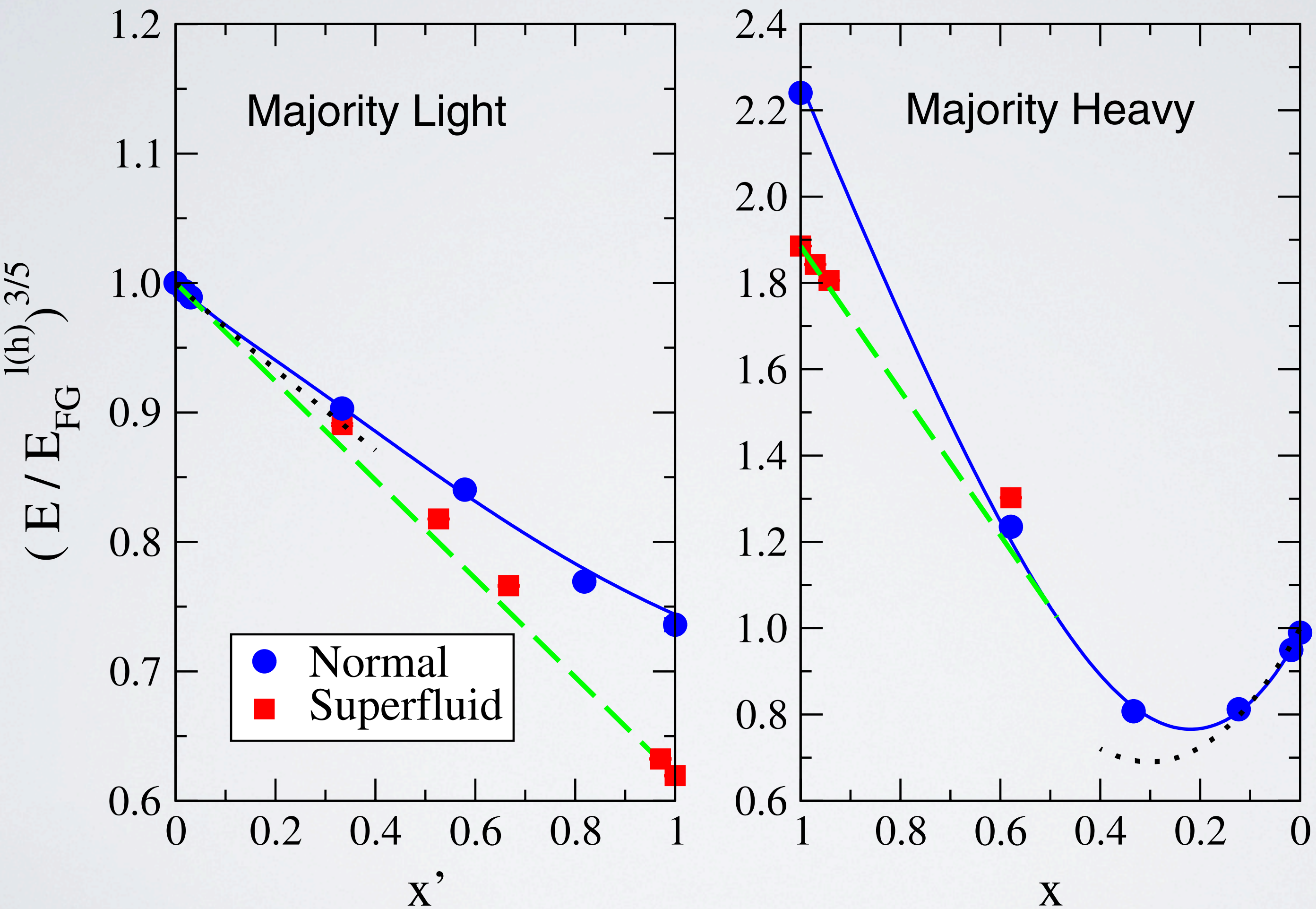
$B(L) = 2.3\ E_F(H)$

effective mass ~ 1.3

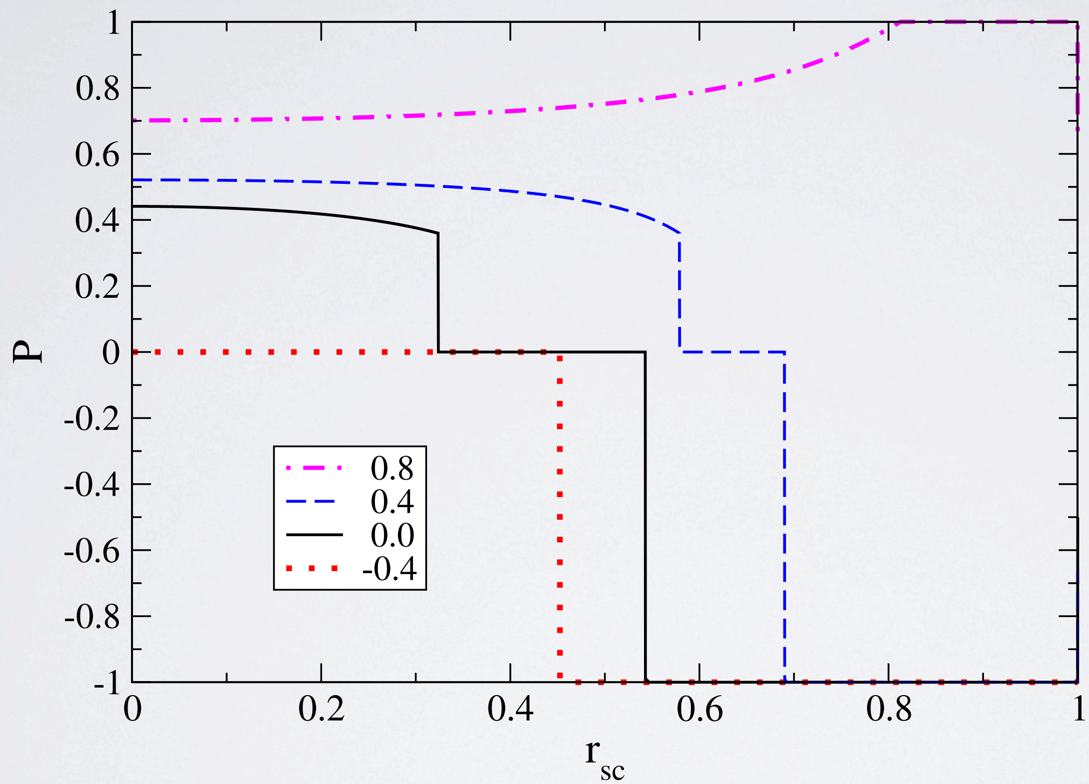
Agreement w/ previous calculations

R. Combescot et al., Phys. Rev. Lett. 98, 180402 (2007)

Polarized Systems and Stable Phases

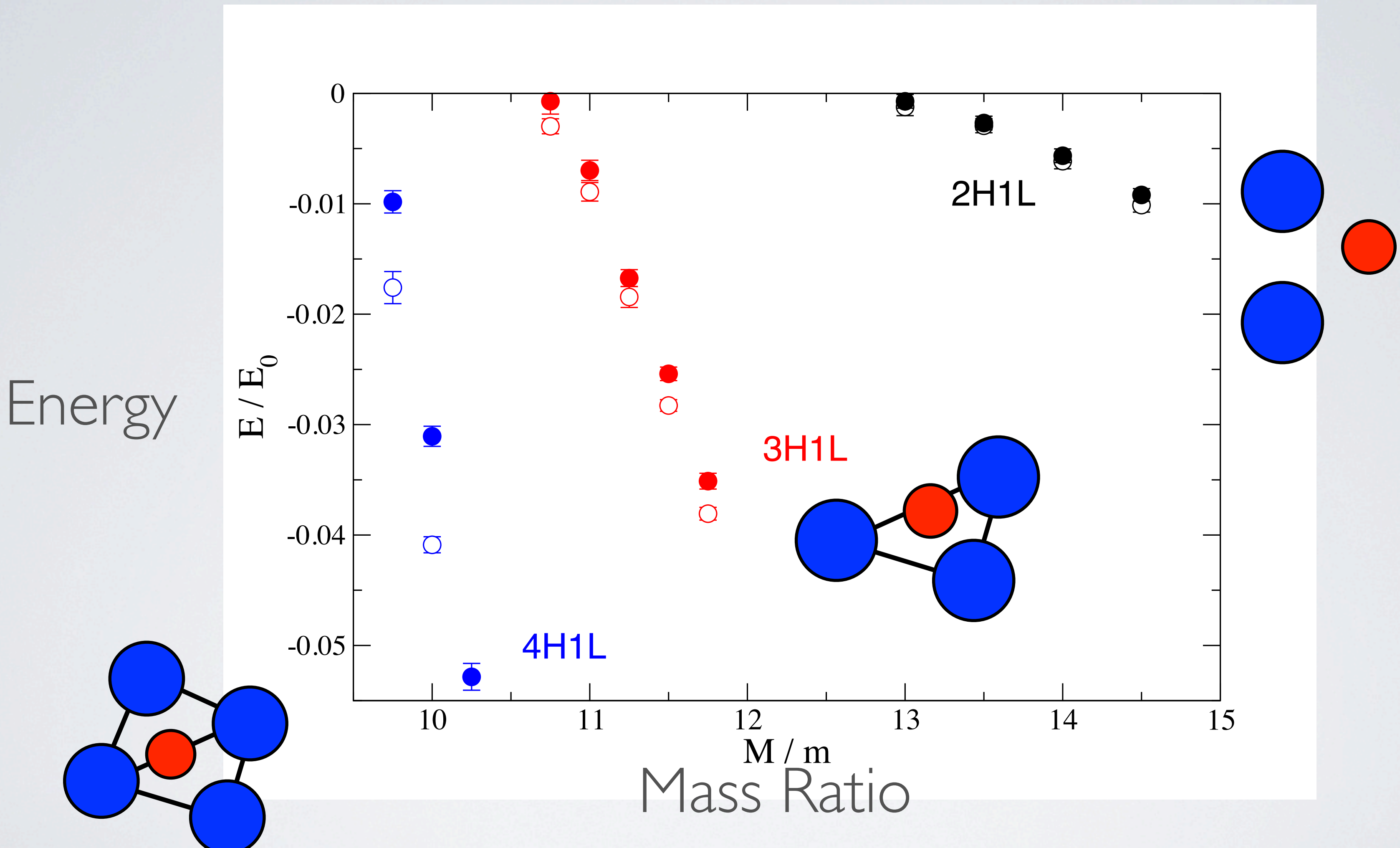


Local Density Approximation for Harmonic Trap



Polarization versus radius for different population imbalances

Non-Universal Behavior N-heavy 1-light at Unitarity



open: gaussian, closed: cosh, ... + 3 body forces,...

heavier systems (eg. 5H1L) less bound

Future Possibilities

'Exotic' Superfluid States
Mixed (or other) Dimensions
Static Response & Finite Systems
Dynamic (RF) Response
Multiple Species
.....